



UNDERSTANDING THE MATHEMATICAL PROOF COMPREHENSION OF TERTIARY MATHEMATICS EDUCATION STUDENTS: A CONVERGENT- PARALLEL DESIGN

Rhea Lee Kyle A. Edio¹, Desiree M. Glenogo², Gemar B. Magbutong^{3*},
and Khif Muamar Miranda⁴

^{1,2}Student Researcher, Davao de Oro State College, Davao de Oro, Philippines

^{3,4}Teacher Education Department, Davao de Oro State College, Davao de Oro, Philippines

*Corresponding Author: gemar.magbutong@ddosc.edu.ph

Abstract

Understanding mathematical proof remains a persistent challenge among students in mathematics education programs, particularly in developing both conceptual reasoning and confidence in proof-based tasks. Despite its importance in teacher preparation, limited studies in the local context have examined proof comprehension using an integrated cognitive and affective perspective. This study investigated the level of mathematical proof comprehension among Tertiary Mathematics education students and examined differences based on year level and academic background using a convergent parallel mixed-methods design. Quantitative findings revealed that students demonstrated a moderate level of proof comprehension, with variations observed across year levels but not across academic backgrounds. Qualitative results revealed key themes in students' lived experiences, including frustration, anxiety, confusion, self-doubt, and feelings of inadequacy, alongside the development of determination and a growth mindset as adaptive responses to difficulty. Students also employed coping strategies such as strategic learning approaches and effective study strategies, including breaking down proofs, repeated practice, and seeking peer support to enhance understanding. The integration of findings indicates that proof comprehension is influenced by the interaction of cognitive, affective, and strategic factors, underscoring the need for structured instructional support and learner-centered approaches in proof-based learning. These findings provide practical insights for improving instruction in teacher education programs.

Keywords: Mathematics education, Mathematical proofs comprehension, Tertiary Mathematics education students, convergent- parallel design, Davao de Oro

Date received: February 23, 2026
Date accepted: May 7, 2026

Date revised: April 14, 2026
Similarity Index: 6%

How to cite:

Edio, R. L. K., Glenogo, D., Magbutong, G., & Miranda, K. M. (2026). Understanding the mathematical proof comprehension of tertiary mathematics education students: A convergent- parallel design. *DDOSC Multidisciplinary Research Journal*, 4(1), 105–121. <https://ddosc.edu.ph/2026/05/20/ddoscmrj-v4i1-008/>

INTRODUCTION

Mathematical proof comprehension is a fundamental component of advanced mathematics learning, particularly for students in teacher education programs. However, empirical evidence indicates that many undergraduate students experience difficulty in understanding proofs. For instance, Morali and Filiz (2023) found that pre-service mathematics teachers frequently demonstrate difficulties in evaluating the correctness of theorems and proofs, often relying on rote procedures rather than a deep understanding of proof structures. Meanwhile, Selden and Selden (2015) found that students often struggle to accurately validate and evaluate proofs despite repeated exposure. These findings highlight persistent gaps in students' deep understanding of mathematical proofs.

While exposure to mathematical proofs is a central component of higher-level mathematics instruction, it does not necessarily lead to meaningful understanding. Many students struggle to recognize the structure, purpose, and logical progression of proofs, often focusing on procedural steps rather than underlying concepts (Laugwitz et al., 2025). This difficulty is further compounded by instructional practices that emphasize rapid content delivery with limited conceptual scaffolding, which may hinder students' ability to internalize proof ideas (Neuhaus & Rach, 2019). Consequently, students tend to rely on memorization instead of engaging in deeper reasoning and justification, limiting their development of genuine proof comprehension.

Beyond cognitive challenges, affective factors also influence students' engagement with proofs. Learners frequently experience anxiety, frustration, and lack of confidence when dealing with proof-based tasks, which may reduce motivation and persistence (Rosenroth et al., 2023). These emotional responses can lead to avoidance behaviors and negatively affect learning outcomes. However, some students demonstrate adaptive responses, such as persistence and the development of a growth mindset, which support continued engagement and improvement in proof comprehension.

Despite extensive international literature, empirical studies on mathematical proof comprehension within Philippine teacher education institutions remain limited. Existing research in the local context often focuses on quantitative performance measures, offering minimal insight into students' lived experiences and emotional responses. This gap highlights the need for a more comprehensive approach that integrates both cognitive and affective dimensions of proof comprehension.

Anchored on this gap, the present study employed a convergent parallel mixed-methods design to examine the mathematical proof comprehension of tertiary Mathematics education students. Specifically, it aimed to determine the level of proof comprehension, examine differences based on selected profile variables such as year level and academic background, and explore students' lived experiences and coping strategies in understanding mathematical proofs. By integrating quantitative and qualitative findings, the study provides a more holistic understanding of how tertiary mathematics education students engage with proof-based learning and offers insights for improving instructional practices in mathematics education.

Research Questions and Significance of the Study

This study aimed to examine the mathematical proof comprehension of tertiary Mathematics education students using a convergent parallel mixed-methods approach. Specifically, it sought to answer the following research questions:

1. What is the profile of the respondents?
2. What is the level of mathematical proof comprehension among tertiary Mathematics students?
3. Is there a significant difference in the mathematical proof comprehension of students when grouped according to:
 - 3.1 Senior High School (SHS) strand; and
 - 3.2 Year level?
4. What are the lived experiences of tertiary Mathematics students who encounter difficulties in comprehending mathematical proofs?
5. How do students cope with the challenges they experience in understanding mathematical proofs?

6. To what extent do the quantitative findings corroborate the qualitative results?

This study contributes to mathematics education by providing empirical evidence on the cognitive and emotional dimensions of mathematical proof comprehension among tertiary Mathematics students. The findings offer practical insights for students and mathematics teachers by identifying common challenges, coping strategies, and year-level differences that can inform more responsive and supportive instructional practices. At the institutional level, the results may guide curriculum planning and the development of targeted interventions to strengthen proof-based instruction in teacher education programs. Moreover, this study adds to the limited local literature on proof comprehension by employing a mixed-methods approach, offering future researchers a context-specific reference for investigating mathematical reasoning and instructional improvement in tertiary education.

METHOD

Research Design

This study employed a convergent parallel mixed-methods research design to investigate the mathematical proof comprehension of tertiary Mathematics education students. The mixed-methods approach was selected to allow a comprehensive examination of proof comprehension by integrating numerical trends with students' experiential insights. According to Creswell and Plano Clark (2011), convergent designs are appropriate when quantitative and qualitative data are collected simultaneously and given equal importance to address a single research problem. In this study, the quantitative component focused on determining the level of proof comprehension and identifying differences across student profiles, while the qualitative component explored students' experiences, challenges, and perceptions related to understanding mathematical proofs.

Both quantitative and qualitative data were gathered during the same phase of the research and analyzed independently using appropriate statistical and thematic procedures. The results from the two data strands were then merged during the interpretation stage to identify points of convergence, complementarity, or divergence. This integration strengthened the validity and credibility of the findings through triangulation and provided a more holistic understanding of proof comprehension among pre-service mathematics teachers. By employing a convergent parallel design, the study captured both performance-based outcomes and affective dimensions of learning, which are essential in informing instructional strategies and intervention development.

Research Locale and Respondents/Participants

The study was conducted in one of the public tertiary education institutions in the Province of Davao de Oro, Philippines. The institution offers a Bachelor of Secondary Education program with a specialization in Mathematics, where students are exposed to proof-based courses such as Logic and Set Theory. This setting provided an appropriate context for examining students' mathematical proof comprehension within a teacher education program. For the quantitative component, 100 BSED Mathematics students from first year to third year were selected to assess their level of mathematical proof comprehension. In the qualitative component, fourteen (14) students were purposively chosen based on their demonstrated difficulties in understanding mathematical proofs, with seven (7) participating in in-depth interviews and seven (7) in focus group discussions. This combination of respondents and participants ensured that the study captured both general patterns of proof comprehension and in-depth insights into students' lived experiences, thereby supporting the mixed-methods design of the research.

Research Instrument

Two instruments were used to collect quantitative and qualitative data. For the quantitative component, a validated multiple-choice questionnaire on mathematical proof comprehension was utilized to assess students' understanding of direct proofs by mathematical induction. The instrument measured key dimensions such as identification of given statements, logical sequencing, interpretation of symbolic expressions, and evaluation of proof validity. Content validity was established through expert review, and the instrument demonstrated acceptable internal consistency based on pilot testing (Cronbach's $\alpha = 0.763$). The interpretation of students' scores is presented in Table 1.

Table 1. Table of Interpretation for Mathematical Proof Comprehension

Range of Means	Descriptive Level	Interpretation
0.00 – 3.33	Low	Indicates limited level of understanding and performance
3.34 – 6.67	Moderate	Indicates an average level of understanding and performance
6.68 – 10.00	High	Indicates a high level of understanding and performance

For the qualitative component, a semi-structured interview guide was employed to explore students' lived experiences, emotional responses, and coping strategies in comprehending mathematical proofs. The guide underwent expert validation and allowed flexibility for probing to obtain in-depth responses. Together, these instruments ensured the systematic collection of both reliable quantitative data and rich qualitative insights aligned with the objectives of the study.

Data Gathering Procedure

Prior to data collection, ethical clearance and necessary institutional approvals were secured. Participants were informed of the purpose of the study, and informed consent was obtained to ensure voluntary participation and confidentiality.

Data were collected concurrently in accordance with the convergent parallel mixed-methods design. For the quantitative component, the proof comprehension questionnaire was administered to the respondents during scheduled sessions with clear instructions provided. For the qualitative component, selected participants took part in in-depth interviews and focus group discussions using a semi-structured interview guide. With consent, the sessions were audio-recorded and subsequently transcribed for analysis.

Data Analysis

The collected data were analyzed using appropriate quantitative and qualitative techniques aligned with the study objectives.

Quantitative data were analyzed using descriptive and nonparametric statistical techniques due to non-normal distribution. Weighted mean and standard deviation were used to describe the level of mathematical proof comprehension. The Kruskal–Wallis test was applied to determine significant differences across year levels, while the Chi-square test was used to examine differences based on academic background. A significance level of 0.05 was adopted.

Qualitative data from interviews and focus group discussions were analyzed using thematic analysis, which involved coding, categorization, and identification of recurring themes related to students' experiences and coping strategies. The quantitative and qualitative findings were then integrated during the interpretation phase to identify convergence and complementarity.

Ethical Considerations

This study was conducted in accordance with established ethical standards for educational research. Ethical clearance was obtained from the Davao de Oro State College Research Ethics Committee prior to data collection. Participation was voluntary, and informed consent was secured from all respondents. Confidentiality and anonymity were ensured throughout the study.

RESULTS AND DISCUSSION

Demographic Profile of the Respondents

The demographic profile describes the distribution of respondents based on selected background variables, specifically year level and Senior High School (SHS) strand. These variables provide contextual information that may influence students' mathematical proof comprehension and help in interpreting variations in performance. Table 2 presents the demographic profile of the respondents.

Table 2. Demographic Profile of Respondents ($N = 100$)

Variable	Category	F	%
Year Level	1st Year	20	20
	2nd Year	44	44
	3rd Year	36	36
	Total	100	100
	SHS Strand	STEM	22
	ABM	29	29
	GAS	23	23
	HUMSS	11	11
	TVL	15	15
	Total	100	100

The results show that most respondents were second-year students (44%), followed by third-year (36%) and first-year students (20%), indicating that the sample is largely composed of learners with moderate exposure to tertiary mathematics. In terms of SHS strand, the largest proportion of respondents graduated from ABM (29%), followed by GAS (23%) and STEM (22%), while fewer completed HUMSS (11%) and TVL (15%). This distribution suggests that respondents come from diverse academic backgrounds, with a notable representation from non-STEM strands.

The variation in academic background implies differences in prior exposure to mathematical concepts, which may influence students' initial readiness for proof-based learning. However, Lestyanto et al. (2022) found that proof comprehension is influenced by both internal and external factors, including content knowledge, experience with proof tasks, teaching strategies, textbook language, and algebraic skills. The study further emphasizes the need to strengthen conceptual understanding alongside procedural knowledge, highlighting the importance of structured instructional support in teacher education programs.

Status of Students' Mathematical Proof Comprehension

Mathematical proof comprehension refers to students' ability to understand, interpret, and evaluate the logical structure and validity of mathematical arguments. It involves recognizing relationships between statements, interpreting symbolic expressions, and following the sequence of reasoning within a proof. Table 3 presents the level of students' mathematical proof comprehension.

Table 3. Level of Students' Mathematical Proof Comprehension

Variable	Mean	SD	Description
Mathematical Proof Comprehension	5.67	1.91	Moderate

The results indicate that students demonstrated a moderate level of proof comprehension ($M = 5.67$, $SD = 1.91$), suggesting an average level of understanding and performance. This implies that while students possess foundational knowledge, they still encounter difficulties in comprehending mathematical proofs, with variability indicating differences in understanding. Although normality tests indicated non-normal distribution, the use of mean and standard deviation is justified by the near-normal distribution reflected in acceptable skewness (0.15) and kurtosis (-0.28) values.

This level of performance is consistent with findings from recent studies on proof comprehension among pre-service teachers. Mohamad Waluyo and Vidákovich (2021) reported that students obtained scores ranging from 49% to 54%, indicating a moderate level of comprehension with only slight improvement across year levels, suggesting that understanding often remains at a developing stage. Similarly, difficulties in proof comprehension persist in tertiary education due to multiple factors, including prior mathematical experience, conceptual understanding, instructional strategies, and clarity of learning materials (Lestyanto et al., 2022). Students' performance in proof-related tasks is also influenced by psychological and instructional factors (Sabanal et al., 2024), as well as challenges in transitioning to the abstract and formal nature of tertiary mathematics, which require improved scaffolding and instructional support (Laugwitz et al., 2025). Moreover, foundational gaps in mathematical knowledge, including proof skills, continue to contribute to students' difficulties,

emphasizing the need to strengthen conceptual understanding within the curriculum (Saha et al., 2024).

Difference in Mathematical Proof Comprehension of Tertiary Education Students in terms of SHS Strand and Year Level.

Proof comprehension, when examined across profile variables such as SHS strand and year level, provides insight into whether students' background and academic progression influence their ability to understand and construct proofs. Table 4 presents the differences in mathematical proof comprehension when grouped according to these variables.

Table 4. Differences in Mathematical Proof Comprehension by Profile Variables

Variable	Chi-square	df	p-value	Remarks
SHS Strand	7.197	4	0.120	Not Significant
Year Level	42.209	2	< 0.001	Significant

These findings are consistent with recent studies on proof comprehension in tertiary education. Waluyo and Vidákovich (2021) reported that first-year students obtained the lowest scores, with significant improvement observed in higher year levels, although gains plateaued beyond the second year. Similarly, studies on students' perceptions of mathematical proof indicate that understanding matures with increased exposure to proof-based instruction and adaptation to the formal nature of higher mathematics (Laugwitz et al., 2025; Kinnear & Inglis, 2026). These results support the present finding that proof comprehension improves as students progress academically.

However, evidence regarding prior academic background remains mixed. While the current study found no significant difference based on SHS strand, Perez et al. (2024) reported that students from STEM backgrounds outperformed those from other strands in general mathematics achievement, suggesting that prior preparation may influence performance in some contexts. In contrast, studies such as Mukuka and Tatira (2025) emphasize that difficulties in proof comprehension persist regardless of background, particularly in more complex proof types. Overall, these findings suggest that although prior academic preparation may contribute to mathematical performance, proof comprehension is more consistently shaped by instructional exposure, cognitive engagement, and experience with proof-based tasks in tertiary education.

Lived Experiences in Comprehending Mathematical Proofs

Students' lived experiences in comprehending mathematical proofs provide insight into the affective dimensions that accompany cognitive challenges in proof-based learning. Analysis of interview and focus group data revealed several recurring themes that describe how students experience and respond to difficulties in understanding proofs.

Figure 1 presents the emergent themes, namely: (1) struggling with frustration and anxiety, (2) losing motivation and confidence, (3) confronting feelings of inadequacy, (4) questioning one's competence, (5) developing determination and a growth mindset, and (6) navigating confusion.

Struggling with Frustration and Anxiety

The students' lived experiences in comprehending mathematical proofs were strongly characterized by frustration and anxiety. Participants described emotional distress when they encountered difficulty understanding proofs, particularly when confusion arose while following proof steps or interpreting arguments. These emotional reactions often accompanied moments when students were unable to make sense of the logical flow of proofs.

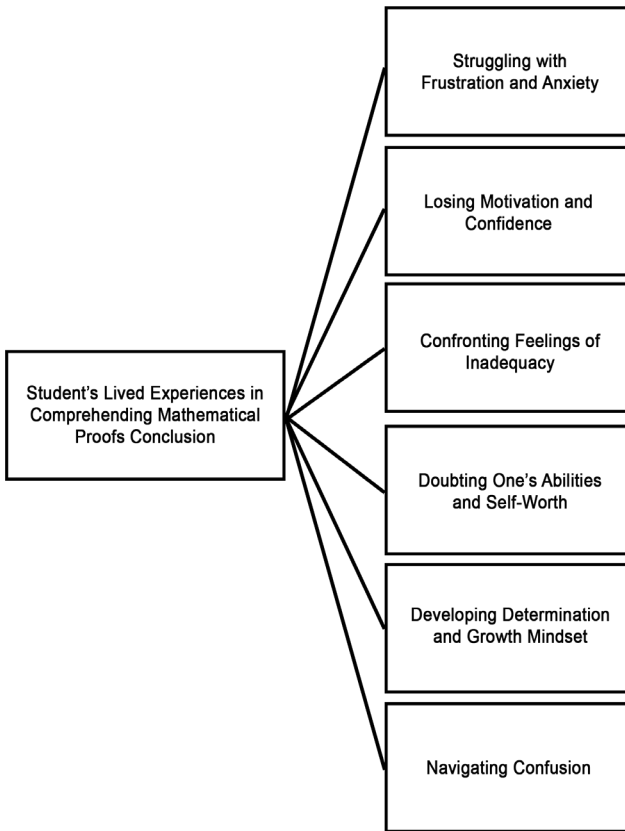


Figure 1. Students' Lived Experiences in Comprehending Mathematical Proofs

Participant 13 shared a common sentiment regarding the frustration experienced when encountering complex mathematical proofs.

"...when I find proofs difficult to understand, I feel frustrated due to their complexity."—IDI 6

Similarly, Participant 10 reported confusion and frustration when dealing with proofs, particularly because of their complexity.

"... I experience both confusion and frustration due to the complexity of mathematical proofs."—FGD 3

These findings are supported by existing literature indicating that students commonly experience frustration and anxiety when engaging in proof-based tasks. Students often feel "stuck" when they cannot identify logical connections between steps, leading to difficulty in reconstructing arguments and understanding proof structure (Riegel, 2021). Similarly, studies report that students struggle to grasp the overall coherence of proofs, resulting in confusion and anxiety when attempting to justify their reasoning (Tapo & Rudhito, 2025; Weber, 2015).

Moreover, the nature of mathematical proof as a socially and academically defined standard may intensify students' anxiety, as expectations of rigor are not always explicit (Granville, 2023). Negative emotional responses, including frustration and mental blockage, have also been shown

to hinder engagement with abstract mathematical tasks, particularly when students draw from prior negative learning experiences (Feraren et al., 2025). These findings highlight the need to address both cognitive and affective factors in supporting students' proof comprehension.

Losing Motivation and Confidence

Students' difficulty in comprehending mathematical proofs was also associated with decreased motivation and reduced confidence in learning mathematics. Participants described how repeated struggles with understanding proofs gradually weakened their interest and enthusiasm for engaging in proof-based tasks. As students encountered persistent confusion and difficulty, their confidence in their mathematical abilities diminished, leading to reduced motivation to participate actively in proof-related learning activities.

Participant 1 further elaborated that those emotional responses and mindset influenced motivation and confidence when engaging with challenging mathematical tasks

"...when I approach mathematics with a positive and open mindset, I feel more motivated to engage with challenging problems; however, when I experience anxiety or frustration, my motivation decreases and my confidence is negatively affected."—FGD 1

In addition, Participant 12 noted that emotional experiences influenced motivation and confidence in learning mathematics.

"... emotional responses significantly influence my motivation and confidence in learning mathematics. Negative emotions, such as fear and frustration, hinder my motivation and contribute to decreased interest and self-doubt in the subject."—FGD 5

These findings are supported by literature indicating that repeated difficulties in proof comprehension can erode students' confidence and motivation. Students who struggle to construct or understand proofs often begin to doubt their mathematical ability, reducing their willingness to engage in new tasks and leading to avoidance of proof-related activities (Bendol & Dalayap, 2025; Aguilar, 2021). Persistent challenges, coupled with anxiety and fear of failure, further weaken motivation, particularly when students attribute difficulties to low ability rather than temporary setbacks (Panerio & Delideli, 2025). These negative experiences may extend to broader attitudes toward mathematics, reinforcing a cycle of low confidence, reduced persistence, and disengagement. Given that confidence is a strong predictor of motivation and engagement, these findings underscore the importance of instructional practices that build self-efficacy through supportive feedback, scaffolded learning, and opportunities for success in proof-based tasks (Bendol & Dalayap, 2025).

Confronting Feelings of Inadequacy

Students' experiences in comprehending mathematical proofs were also marked by feelings of inadequacy, particularly in relation to their self-perception as learners of mathematics. Participants described how persistent difficulty in understanding proofs led them to view themselves as incapable of grasping mathematical concepts.

Participant 4 described how difficulty in understanding mathematical proofs led to feelings of inadequacy and negative self-perceptions about personal ability in mathematics.

"...when I struggle to understand mathematical proofs, I tend to perceive myself as lacking mathematical ability, which leads to feelings of discouragement and frustration."—FGD 4

Further, Participant 2 described how difficulty in comprehending mathematical proofs led to feelings of inadequacy and a perceived inability to grasp mathematical concepts.

“...difficulty in comprehending mathematical proofs is a significant challenge, as I feel unable to fully grasp the concepts, prompting me to seek guidance from my classmates.” —FGD 2

These findings are supported by recent literature indicating that persistent difficulties in proof comprehension are often internalized as personal inadequacy. Students who struggle with constructing and justifying proofs tend to interpret their difficulties as a lack of ability, leading to decreased confidence, anxiety, and negative attitudes toward mathematics (Abd, Algani, 2024). This perception is further reinforced when students compare themselves with peers or instructors, amplifying feelings of inadequacy despite having the capacity to learn (Viholainen et al., 2019). Difficulties in structuring proofs and providing logical justifications also contribute to the belief that one lacks the “right” mathematical ability, resulting in avoidance and disengagement from proof-related tasks (Mercado, 2025). Moreover, when proofs are perceived as complex or reserved for highly capable individuals, students become reluctant to engage due to fear of failure or exposure of their perceived limitations (Laugwitz et al., 2025). Broader evidence likewise shows that declining mathematical performance is often interpreted as personal deficiency, which extends feelings of inadequacy beyond proof comprehension and reduces students’ persistence in challenging mathematical tasks (National Science Board [NSB], 2023).

Doubting One’s Ability and Self-worth

Students’ difficulty in comprehending mathematical proofs also resulted in shaken confidence and self-worth, particularly in relation to their perceived ability to solve mathematical problems. Participants described how repeated struggles with proofs led them to question their competence and problem-solving skills.

Participant 1 described how persistent difficulty in comprehending mathematical proofs led to shaken confidence and heightened self-worth, affecting both academic self-perception and commitment to mathematics.

“...struggling with mathematical proofs has significantly undermined my confidence, leading to intense self-doubt and causing me to question my abilities and suitability as a mathematics student.”
—FGD 1

These findings are supported by recent literature indicating that difficulties in proof comprehension often lead students to question their own competence. Students who repeatedly struggle with proof construction and justification tend to attribute these challenges to a lack of ability, resulting in heightened self-doubt even when they demonstrate partial understanding (Abd, Algani, 2024; Mercado, 2025). This sense of inadequacy is further reinforced when students perceive proofs as tasks suited only for more capable peers, leading them to question their place in mathematics-related programs (Viholainen et al., 2019; Laugwitz et al., 2025). Moreover, broader educational evidence suggests that declines in mathematical performance are frequently interpreted as indicators of low personal competence, which can weaken students’ confidence and persistence in proof-heavy tasks (NSB, 2023). These patterns highlight the need for instructional approaches that explicitly build students’ confidence, normalize difficulty in proof learning, and reinforce the idea that competence in mathematics develops through guided practice and sustained effort.

Developing Determination and a Growth Mindset

Despite the challenges encountered in comprehending mathematical proofs, some students demonstrated determination and a growth mindset as adaptive responses to difficulty. Rather than withdrawing from complex tasks, these students approached challenges as opportunities for learning and improvement, showing persistence in understanding proofs. Their experiences reflect an active effort to overcome confusion and frustration by maintaining focus, exerting effort, and adopting positive learning dispositions toward proof-based tasks.

Participant 3 emphasized how emotional reactions to difficulty can inspire perseverance and determination.

“Emotional responses to challenges in understanding mathematical proofs can foster perseverance and determination, motivating me to overcome these difficulties.”—FGD 3

Similarly, Participant 6 explained that effort and persistence enabled continued engagement despite difficulty.

“I respond by maintaining the belief that I can succeed through sustained effort, continuing to work diligently to correctly execute proofs and understand the underlying concepts.”—FGD 6

These findings are supported by recent literature indicating that determination and growth mindset play a critical role in sustaining students' engagement in proof-based learning. Students who view challenges as opportunities for growth are more likely to persist despite difficulty, reinterpret failure constructively, and remain engaged in abstract mathematical tasks (Riegel, 2021; Rattan et al., 2022). Empirical studies further show that growth mindset, when combined with self-efficacy and intrinsic motivation, enhances students' willingness to exert effort, adapt strategies, and maintain persistence in complex problem-solving contexts (Dong et al., 2023; Xu & Dieckmann, 2025). These findings suggest that students' ability to cope with frustration and develop determination is not merely a personal trait but is supported by cognitive and motivational processes that enable sustained engagement in proof-heavy mathematics.

Navigating Confusion

Proof comprehension emerged as a critical yet challenging skill for students, with confusion being a common experience when engaging with mathematical proofs. Participants described difficulty identifying the starting point of a proof, particularly the hypotheses, and following the logical flow between successive steps. This lack of clear connections often led to feelings of being lost and disoriented, making it difficult for students to grasp the overall argument of a proof. Such confusion hindered students' ability to meaningfully engage with proofs and contributed to frustration during the learning process.

Participants described confusion as an immediate response when encountering difficulties in comprehending mathematical proofs, often accompanied by frustration, worry, and self-doubt, particularly when they could not follow the logical flow or determine how to proceed.

“...when I struggle to comprehend mathematical proofs, I feel frustrated, confused, and experience self-doubt, particularly when I compare my performance with others.”—FGD 4

“...when I encounter difficulties, I tend to feel worried and confused about how to proceed.”—FGD 2

Confusion in mathematical proof comprehension among tertiary education students commonly arises from difficulties in following logical steps, identifying starting points, and connecting abstract ideas, often leading to cognitive disorientation and challenges with symbolic representations (Laugwitz et al., 2025; Lestyanto et al., 2022; Mutodi & Mosimege, 2021). While unresolved confusion can hinder engagement and persistence in proof-based tasks, particularly in abstract domains, research suggests that when appropriately scaffolded through instructional support and active learning strategies, it can become a catalyst for deeper understanding and resilience (Zengilowski & Brown, 2025; Craig et al., 2019). These findings highlight the importance of transforming confusion into a productive learning experience through targeted pedagogical interventions.

Coping Strategies in Comprehending Mathematical Proof

Despite the challenges in comprehending mathematical proofs, students employed coping strategies to manage the cognitive and emotional demands of proof-based learning. Qualitative findings show that students actively addressed confusion, frustration, and self-doubt in order to sustain engagement and improve their understanding of proofs.

The coping strategies identified in this study are organized into two major themes: Applying Strategic Learning Approaches Strategic Learning (STLG), and Utilizing Effective Study Strategies, and. These themes illustrate how students actively responded to challenges in proof comprehension by regulating their learning processes and applying structured study approaches. These coping strategies are further supported by students' development of a growth mindset and persistence, which enable them to sustain engagement and respond adaptively to challenges in proof comprehension.

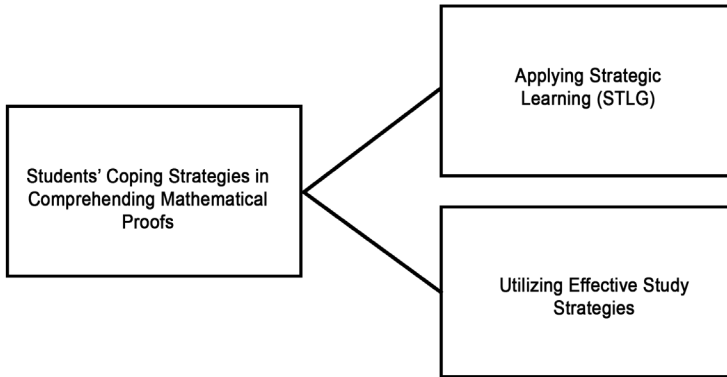


Figure 2. Students' Coping Strategies in Comprehending Mathematical Proofs

Applying Strategic Learning Approaches Strategic Learning

Students demonstrated strategic learning as a primary coping mechanism in comprehending mathematical proofs. Participants emphasized taking deliberate control of their learning process by planning, organizing study time, and approaching proofs systematically. Strategic learning allowed students to actively manage the cognitive demands of proof comprehension rather than reacting passively to difficulty.

Participants described applying strategic approaches to manage the complexity of mathematical proofs, particularly by breaking proofs into smaller, more manageable steps and seeking support from peers or instructors to gain different perspectives.

"...I approach proofs by breaking them into smaller, manageable parts, focusing on understanding each step, and seeking support from classmates or tutors to gain additional insights."—FGD 1

"...when proofs are challenging, I divide them into smaller steps, seek help from peers or teachers, and practice similar problems to improve understanding."—IDI 2, 6

These findings are supported by literature indicating that strategic learning approaches enhance students' ability to manage the complexity of proof-based tasks. Breaking problems into smaller steps, seeking peer support, and engaging in collaborative learning help reduce cognitive load and improve understanding of abstract concepts (Klang et al., 2021). Similarly, explicit strategies for proof comprehension, such as identifying key ideas, monitoring logical steps, and evaluating understanding, enable students to process proofs more effectively rather than treating them as isolated procedures (Weber & Mejia-Ramos, 2013).

Moreover, metacognitive and self-regulated learning strategies, including self-explanation, planning, and monitoring, have been shown to significantly improve students' higher-order thinking and proof comprehension by promoting deeper engagement with mathematical reasoning (Gao et al., 2025; Sommerhoff et al., 2021; Hidayat et al., 2025). These strategies allow learners to actively regulate their thinking, while self-regulation processes further support sustained effort and adaptive learning behaviors in complex mathematical tasks (Aydan & Capa-Aydin, 2025). Overall, these findings highlight that strategic learning is essential in helping students navigate the cognitive demands of proof comprehension and develop more effective problem-solving skills.

Utilizing Effective Study Strategies

Students reported using repeated practice and systematic review as key strategies in comprehending mathematical proofs. They emphasized revisiting proofs multiple times, clarifying unclear steps, and organizing ideas through notes or summaries, which helped reinforce understanding and improve their ability to engage with proof-based tasks.

Participants described using repeated practice and systematic review as key strategies to strengthen their understanding of mathematical proofs.

"...understanding of mathematical proofs is strengthened through repeated practice, reviewing proofs multiple times, revisiting unclear sections, and organizing ideas through notes or summaries."

—FGD 1; IDI 4

These findings are supported by research indicating that repeated practice and systematic review are effective strategies for improving mathematical understanding, particularly in proof-based tasks. Techniques such as spaced and interleaved practice enhance long-term retention and problem-solving performance by encouraging retrieval and deeper processing of abstract concepts (Hartwig & Rohrer, 2025). Similarly, guided note-taking and structured review support students in organizing ideas, revisiting unclear steps, and reinforcing conceptual understanding, which are essential for comprehending mathematical proofs (Feudel & Panse, 2022).

Moreover, broader evidence highlights that cognitive and metacognitive strategies—such as summarizing, connecting prior knowledge, and reflective review—promote higher-order thinking and sustained engagement in complex mathematical tasks (Idris et al., 2024). These strategies enable students to consolidate learning through repeated exposure and active processing, thereby strengthening their ability to understand and apply proof-based reasoning.

Data Integration of Salient Quantitative and Qualitative Findings

The integration of quantitative and qualitative findings provides a comprehensive understanding of students' mathematical proof comprehension, highlighting how cognitive, affective, and strategic factors interact in shaping learning outcomes. This integrated perspective explains variations in performance and students' experiences in engaging with proof-based tasks. Table 5 presents the integrated findings across key aspects of the study, combining statistical results with students' lived experiences and coping strategies.

Table 5. Integrated Findings

Aspect for Focal Point	Quantitative Findings	Qualitative Findings	Nature of Data Integration	Axiological Implications
Overall Proof Comprehension	Moderate level of proof comprehension	Confusion, frustration, and difficulty following logical flow	Convergence: Moderate performance reflects cognitive and emotional barriers	Instruction should address both reasoning skills and affective support

Table 5. *Continued*

Aspect for Focal Point	Quantitative Findings	Qualitative Findings	Nature of Data Integration	Axiological Implications
SHS Strand	No significant difference across strands	Struggles reported across participants regardless of background	Complementarity: Strand does not explain difficulty; program-level factors are more salient	Provide equitable scaffolding for all learners
Year Level	Significant difference; higher-year students performed better	Development of coping strategies and persistence over time	Expansion: Qualitative data explain performance gap through growth and adaptation	Strengthen early-year instructional support
Affective Factors	Moderate comprehension with variability	Anxiety, self-doubt, decreased confidence	Expansion: Emotional factors shape engagement and comprehension	Foster confidence-building and emotion-aware pedagogy
Coping Mechanisms	Not directly measured quantitatively	Strategic learning, effective study strategies, growth mindset	Complementarity: Coping explains sustained engagement despite difficulty	Integrate study regulation and mindset training into curriculum
Instructional Insights	—	Collaborative learning, active participation, time planning, personalized study	Developmental Integration: Student insights inform instructional refinement	Promote interactive, structured, and learner-centered approaches

The results indicate that students demonstrate a moderate level of proof comprehension, which is closely associated with their experiences of confusion, frustration, and difficulty in following logical structures. This suggests that performance is influenced not only by cognitive ability but also by affective factors that shape students' engagement with proofs. Furthermore, the lack of significant differences across SHS strands implies that prior academic background has limited influence on proof comprehension, whereas the significant differences across year levels indicate that understanding improves with increased exposure, experience, and familiarity with proof-based tasks.

Qualitative findings further explain these results by showing that students develop coping strategies, such as breaking down proofs, engaging in repeated practice, and seeking peer support, which help them manage cognitive demands. These strategies are further supported by the development of a growth mindset and persistence, which function as underlying mechanisms that sustain students' engagement and enable them to apply coping strategies effectively in overcoming difficulties in proof comprehension. This aligns with previous studies indicating that proof comprehension is shaped more by instructional experiences and engagement rather than prior academic preparation (Lestyanto et al., 2022; Waluyo & Vidákovich, 2021). Similarly, research highlights that cognitive and affective factors jointly influence students' understanding of proofs in higher education (Sabanal et al., 2024; Laugwitz et al., 2025). However, some studies suggest that prior preparation may influence general mathematics performance, indicating that its effect on proof comprehension may be indirect or context-dependent.

Overall, the integrated findings highlight that improving proof comprehension requires addressing not only students' reasoning skills but also their emotional experiences and strategic learning behaviors. These results contribute to mathematics education by providing evidence that effective instruction should incorporate structured support, explicit teaching of proof strategies, and learning environments that foster confidence, persistence, and active engagement.

CONCLUSION

Mathematical proof comprehension among tertiary mathematics education students is shaped by both cognitive and affective factors, reflecting the complexity of learning proof-based concepts. While students demonstrate a foundational understanding, their ability to fully interpret and justify proofs remains developing. The findings highlight that proof comprehension improves through sustained engagement with proof-based instruction rather than prior academic background, emphasizing the role of structured learning experiences within teacher education programs.

Moreover, students' experiences of confusion, anxiety, and self-doubt, alongside their use of strategic learning approaches and persistence, underscore the importance of addressing both cognitive and emotional dimensions of learning. These insights suggest that effective instruction should incorporate explicit teaching of proof structure, opportunities for collaborative learning, and supportive classroom environments that promote confidence and resilience.

In light of these findings, mathematics educators are encouraged to provide targeted instructional support, particularly in the early stages of proof-based learning, and to integrate strategies that foster both conceptual understanding and positive learning dispositions. These findings further highlight the role of growth mindset and persistence in sustaining students' engagement and enabling them to effectively navigate challenges in proof-based learning. Future research may further examine the effectiveness of specific instructional interventions in enhancing students' proof comprehension across different levels of mathematics education.

Acknowledgement

The authors would like to thank all individuals who contributed to the completion of the study.

Conflict of Interest

The authors declare no conflict of interest.

Ethical Statement

The study obtained clearance from DDOSC-REC with protocol code 221-02-2024.

REFERENCES

- Abd, Algani, A. (2024). Solving mathematics anxiety, lack of confidence and negative attitude with artificial intelligence models: Insights from stakeholders. *Journal for the Mathematics Education and Teaching Practices*, 5(2), 89–100. <https://doi.org/10.5281/zenodo.14599899>
- Aguilar, J. J. (2021). High school students' reasons for disliking mathematics: The intersection between teacher's role and student's emotions, belief and self-efficacy. *International Electronic Journal of Mathematics Education*, 16(3), em0658. <https://doi.org/10.29333/iejme/11294>
- Aydan, S., & Capa-Aydin, Y. (2025). What makes them self-regulated? Self-regulation procedures of academically successful students and key influences. *Acta Psychologica*, 257, 105106. <https://doi.org/10.1016/j.actpsy.2025.105106>
- Bendol, R. L., & Dalayap, R. H., Jr. (2025). Structural equation modelling of factors influencing confidence in mathematics. *ASEAN Journal of Educational Research and Technology*, 4(2), 107–124. <https://ejournal.bumipublikasinusantara.id/index.php/ajert/article/view/641>
- Creswell, J. W., & Plano Clark, V. L. (2011). *Designing and conducting mixed methods research* (2nd ed.). SAGE Publications.

- Dong, L., Jia, X., & Fei, Y. (2023). How growth mindset influences mathematics achievements: A study of Chinese middle school students. *Frontiers in Psychology, 14*, 1148754. <https://doi.org/10.3389/fpsyg.2023.1148754>
- Feraren, A. G. A., Gonzales, B. J. B., & Quillano, G. O., Jr. (2025). Understanding students' mathematical problem-solving difficulties: Teachers' strategies and learners' coping mechanisms. *International Journal of Multidisciplinary Studies in Higher Education, 2*(4), 17–36. DOI: 10.70847/661190
- Gao, H., Evans, T., & Fergusson, A. (2025). Student-generated explanation in undergraduate mathematics and statistics education: A systematic literature review. *International Journal of Mathematical Education in Science and Technology, 56*(12), 2534–2557. <https://doi.org/10.1080/0020739X.2025.2556867>
- Granville, A. (2023). Accepted proofs: Objective truth, or culturally robust. *arXiv preprint arXiv:2305.02115*. Retrieved from <https://arxiv.org/abs/2305.02115>
- Hidayat, R., Mohd Saad, M. R., & Wewe, M. (2025). A meta-analysis of the effect of metacognitive instruction on mathematics achievement. *Cogent Education, 12*(1), 2517510. <https://doi.org/10.1080/2331186X.2025.2517510>
- Kinnear, G., & Inglis, M. (2026). Does understanding moderate aesthetic appraisals of proofs? *The Journal of Mathematical Behavior, 81*, 101284. Retrieved from <https://l1nq.com/hsp9d>
- Klang, N., Karlsson, N., Kilborn, W., Eriksson, P., & Karlberg, M. (2021). Mathematical problem-solving through cooperative learning—The importance of peer acceptance and friendships. *Frontiers in Education, 6*, 710296. <https://doi.org/10.3389/feduc.2021.710296>
- Laugwitz, R., Brereton, N., & Cheng, M. (2025). Undergraduate students' perception of mathematical proof. *Teaching Mathematics and its Applications: An International Journal of the IMA*, hraf017. <https://doi.org/10.1093/teamat/hraf017>
- Lestyanto, L. M., Rahmadani, D., Asmianto, A., & Cahyowati, E. T. D. (2022). The factors influencing students' proof comprehension on mathematical induction. In *2nd National Conference on Mathematics Education 2021 (NaCoME 2021)* (pp. 135–139). Atlantis Press. <https://www.atlantispress.com/proceedings/nacome-21/125972964>
- Mercado, H. (2025). *The role of mathematics teacher-interns' past learning experiences in teaching geometric proofs*. <https://sl1nk.com/mq0jlb>
- Morali, H. S., & Filiz, A. (2023). Incorrect theorems and proofs: An analysis of pre-service mathematics teachers' proof evaluation skills. *Journal of Pedagogical Research, 7*(3), 248–262. <https://doi.org/10.33902/JPR.202318840>
- Mukuka, A., & Tatira, B. (2025). Analysis of pre-service mathematics teachers' proof comprehension through Toulmin's argumentation model. *Journal on Mathematics Education, 16*(1), 111–130. <https://doi.org/10.22342/jme.v16i1.pp111-130>
- Mutodi, P., & Mosimege, M. (2021). Learning mathematical symbolization: Conceptual challenges and instructional strategies in secondary schools. *Bolema: Boletim de Educação Matemática, 35*, 1180–1199. <https://doi.org/10.1590/1980-4415v35n70a29>
- National Science Board. (2023, October 26). *COVID-19 pandemic exacerbated widening gap and overall decline in U.S. mathematics performance*. National Science Foundation. <https://www.nsf.gov/nsb/updates/covid-19-pandemic-exacerbated-widening-gap-overall-decline>

- Neuhaus, S., & Rach, S. (2019). Proof comprehension of undergraduate students and the relation to individual characteristics. In *Eleventh Congress of the European Society for Research in Mathematics Education* (No. 31). Freudenthal Group; Freudenthal Institute; ERME. https://hal.science/hal-02398493v1/file/TWG01_31_GS_AMR.pdf
- Panerio, A. M. S., & Delideli, J. A. (2025). Geometric proof struggles on academic achievements in mathematics among third year college students. *International Journal for Multidisciplinary Research*, 7(2), 1–13. <https://www.ijfmr.com/papers/2025/2/38443.pdf>
- Perez, K., et al. (2024). Senior high school strand influence on mathematical competence. *International Multidisciplinary Journal of Research for Innovation, Sustainability, and Excellence*, 1(12). <https://risejournals.org/index.php/imjrise/article/download/888/1257/2640>
- Riegel, K. (2021). Frustration in mathematical problem-solving: A systematic review of research. *STEM Education*, 1(3), 157–169. <https://doi.org/10.3934/steme.2021012>
- Sabanal, D., Gako, M., Torre, H. D., Sabanal, J., So, R. B., Bacal, J. B., ... & Goles, N. (2024). Predictive model for college students' performance in higher mathematics. *Social Sciences & Humanities Open*, 10, 101134.
- Saha, M., Islam, S., Akhi, A. A., & Saha, G. (2024). Factors affecting success and failure in higher education mathematics: Students' and teachers' perspectives. *Heliyon*, 10(7).
- Sarrasin, K., Dieckmann, J. A., et al. (2023). How growth mindset influences mathematics achievements: A multi-construct model of mindset, motivation, and self-efficacy. *Frontiers in Psychology*, 14, Article 10086334. <https://doi.org/10.3389/fpsyg.2023.10086334>
- Selden, A., & Selden, J. (2017). A comparison of proof comprehension, proof construction, proof validation and proof evaluation. In *Proceedings of the Conference on Didactics of Mathematics in Higher Education as a Scientific Discipline* (pp. 339–345). <https://sl1nk.com/83b75sw>
- Sommerhoff, D., Kollar, I., & Ufer, S. (2021). Supporting mathematical argumentation and proof skills: Comparing the effectiveness of a sequential and a concurrent instructional approach to support resource-based cognitive skills. *Frontiers in Psychology*, 11, 572165. <https://pmc.ncbi.nlm.nih.gov/articles/PMC7858258/>
- Sullins, J., Console, K., Denton, R., Henrichson, C., & Barber, S. (2019). Not all confusion is productive: An investigation into confusion induction methods and their impact on learning. *International Journal of Learning Technology*, 14(4), 288–303. <https://sl1nk.com/zmve4pi>
- Tañola, M. D., & Lomibao, L. S. (2024). Understanding how students learn mathematics: A systematic literature review of contemporary learning strategies in mathematics education post-2020. *Journal of Innovations in Teaching and Learning*, 4(1), 66–75. <https://doi.org/10.12691/jitl-4-1-11>
- Tapo, M. M., & Rudhito, M. A. (2025). Mathematical proof difficulties among mathematics education students: A cognitive perspective. *International Journal of Advanced Education and Research*, 10(3), 12–20. <https://alieducationjournal.com/archives/2025/vol10/issue3/10055>
- Viholainen, A., Tossavainen, T., Viitala, H., & Johansson, M. (2019). University mathematics students' self-efficacy beliefs about proof and proving. *LUMAT: International Journal on Math, Science and Technology Education*, 7(1), 148–164. <https://doi.org/10.31129/LUMAT.7.1.406>
- Waluyo, M., & Vidákovich, T. (2021). Assessment of students' mathematical proof comprehension: Gender and year level background. *Journal of Physics: Conference Series*, 1776, 012017. <https://doi.org/10.1088/1742-6596/1776/1/012017>

- Weber, K. (2015). Effective proof reading strategies for comprehending mathematical proofs. *International Journal of Research in Undergraduate Mathematics Education*, 1(3), 289–314. <https://doi.org/10.1007/s40753-015-0011-0>
- Weber, K., & Mejia-Ramos, J. P. (2013). Effective but underused strategies for proof comprehension. *North American Chapter of the International Group for the Psychology of Mathematics Education*. <https://files.eric.ed.gov/fulltext/ED584498.pdf>
- Xu, K. M., Leferink, J., & Wijnia, L. (2025). A review of the relationship between student growth mindset and self-regulated learning. *Frontiers in Education*, 10, 1539639. <https://doi.org/10.3389/educ.2025.1539639>
- Xu, X., & Dieckmann, J. A. (2025). Differentiating mathematical mindset, growth mindset, and self-efficacy through intervention research: A neuroplasticity approach. *Frontiers in Psychology*, 16, Article 1598817. <https://doi.org/10.3389/fpsyg.2025.1598817>
- Zengilowski, A., & Brown, K. E. (2025). How instructors can respond to undergraduates' confusion: Examining students' experiences prior to and after the onset of the COVID-19 pandemic. *Learning in Context*, 2(1–2), 100014. <https://doi.org/10.1016/j.lecon.2025.100014>